Consider the statement **SCORE: ____ / 15 PTS**

P → Q

"Thrashing occurs only if the working set is too small."

(Write your final answers in complete sentences without using any symbols or variables.)

[a] Write the contrapositive of the statement, **using "if / then"**.

IF THE WORKING SET IS NOT TOO SMALL, THEN THRASHING DOES NOT OCCUR

[b] Write a logically equivalent statement using "is necessary for", **without using "if"**.

THE WORKING SET BEING TOO SMALL IS NECESSARY FOR THRASHING TO OCCUR

Write the following statement **informally**. **Avoid ambiguous language.** **SCORE: ____ / 10 PTS**
Your answer should NOT use the phrases "for all", "for every", "for each", "for any", "such that", "there exists".

$\forall r \in P : \exists t \in E, \sim V(r, t)$ where P = set of all people, E = set of all countries in Europe, and $V(r, t)$ = "r has visited t"

NOBODY HAS VISITED EVERY SINGLE COUNTRY IN EUROPE

Write the following statement symbolically, using **TWO** variables. **State clearly the domains and predicate.** **SCORE: ____ / 10 PTS**

"There is a reciprocal for every natural number."

$D = \mathbb{Z}^+ = \{\text{NATURAL NUMBERS}\}$

$P(x, y) =$ "y IS THE RECIPROCAL OF x" OR "$xy = 1$"

$\forall x \in D, \exists y \in \mathbb{R} : P(x, y)$

Consider the statement **SCORE: ____ / 15 PTS**

"Students on the honor roll receive priority registration."

[a] Write the argument symbolically, using **TWO** predicates. **State clearly the domain and predicates.**

$D = \{\text{STUDENTS}\}$

$P(x) =$ "x IS ON THE HONOR ROLL"

$Q(x) =$ "x RECEIVES PRIORITY REGISTRATION"

$\forall x \in D, P(x) \rightarrow Q(x)$

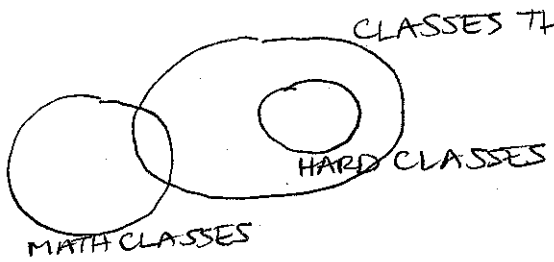
[b] Write the negation of the statement informally.
Your answer should NOT use the phrases "for all", "for every", "for each", "for any", "such that", "there exists".

$\exists x \in D : P(x) \wedge \sim Q(x)$

SOME STUDENT ON THE HONOR ROLL DOES NOT RECEIVE PRIORITY REGISTRATION

Indicate whether the following argument is valid or invalid. **Support your answer using a diagram.** **SCORE: ____ / 10 PTS**

Some math classes require a lot of work.
All hard classes require a lot of work.
Therefore, some math classes are hard classes.



INVALID

Prove that the following argument is valid using the Rules of Inference. **SCORE: ____ / 25 PTS**
Give the reason for each step as shown in lecture.

Do NOT rewrite any of the hypotheses using logical equivalences.

$q \rightarrow \sim s$
$p \rightarrow r$
$w \rightarrow \sim r$
$\sim s \rightarrow r$
$p \vee q$
$\therefore \sim w$

$q \rightarrow \sim s$ GIVEN
$\sim s \rightarrow r$ GIVEN
$\therefore q \rightarrow r$ TRAN
$p \rightarrow r$ GIVEN
$p \vee q$ GIVEN
$\therefore r$ CASE
$w \rightarrow \sim r$ GIVEN
$\therefore \sim w$ MT

Determine if $(p \oplus q) \vee \sim p \equiv (p \leftrightarrow q) \rightarrow \sim q$. Show proper justification & state your final answer clearly. **SCORE: ____ / 20 PTS**

p	q	$p \oplus q$	$(p \oplus q) \vee \sim p$	$p \leftrightarrow q$	$(p \leftrightarrow q) \rightarrow \sim q$
T	T	F	F	T	F
T	F	T	T	F	T
F	T	T	T	F	T
F	F	F	T	T	T

↑ SAME VALUES ↑

SO, LOGICALLY EQUIVALENT

Let $A = \{-2, 1, 3\}$ and $B = \{-2, 0, 2\}$. **SCORE: ____ / 25 PTS**

Let $P(x, y)$ be the predicate "$x^2 + y$ is a multiple of 3" with domain $A \times B$ (ie. $x \in A$ and $y \in B$).

Determine if the statement "$\forall x \in A, \exists y \in B :, P(x, y)$" is true or false.

Justify your answer as shown in lecture. Use as few examples/counterexamples as you need.

$x = -2$ $\exists y \in B : (-2)^2 + y$ IS A MULTIPLE OF 3 TRUE $y = 2$

$x = 1$ $\exists y \in B : 1^2 + y$ IS A MULTIPLE OF 3 TRUE $y = 2$

$x = 3$ $\exists y \in B : 3^2 + y$ IS A MULTIPLE OF 3 TRUE $y = 0$

THE STATEMENT IS TRUE

Write the **formal definition** of a function used in discrete math. **Use correct English.** **SCORE: ____ / 10 PTS**
You may symbolic logic and set notation, if you use it correctly.

A RELATION R FROM SET A TO SET B IS A FUNCTION IFF

① $\forall x \in A, \exists y \in B : xRy$ AND

② $\forall x \in A, \forall y \in B, \forall z \in B, (xRy \wedge xRz) \rightarrow y = z$

Fill in the blanks. Your answers must be in English, **not symbols**. **SCORE: ____ / 10 PTS**

[a] $r \vee s$ is read as "r or s", and is called the DISJUNCTION OF r AND s.

[b] $A \times B$ is read as "A cross B", and is called the CARTESIAN PRODUCT OF A AND B.

[c] In the conditional "$k \rightarrow m$",

m is called the CONCLUSION and k is called the HYPOTHESIS.

[d] R is a **proper** subset of Q if and only if

EVERY ELEMENT OF R IS AN ELEMENT OF Q AND THERE IS AN ELEMENT OF Q THAT IS NOT IN R.

[e] The argument

"Jennifer will enroll at Stanford (P) in the fall only if she receives a scholarship from Stanford (Q).

Jennifer receives a scholarship from Stanford (Q).

Therefore, Jennifer will enroll at Stanford (P) in the fall"

is an example of CONVERSE ERROR.